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CP Violation and the CKM Angle γ from Angular Distributions of Untagged B_s Decays Governed by $\bar{\mathbf{b}} \to \bar{c}u\bar{s}$

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CP violation and the CKM angle γ from angular distributions of untagged B_s decays governed by $\bar{b} \to \bar{c}u\bar{s}$

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Abstract

We demonstrate that time-dependent studies of angular distributions for B_s decays caused by $\bar{b} \to \bar{c}u\bar{s}$ quark-level transitions extract *cleanly* and *model-independently* the CKM angle γ. This CKM angle could be cleanly determined from *untagged* B_s decays alone, if the lifetime difference between the B_s mass eigenstates B_s^L and B_s^H is sizable. The time-dependences for the relevant *tagged* and *untagged* observables are given both in a general notation and in terms of linear polarization states and should exhibit large CP-violating effects. These observables may furthermore provide insights into the hadronization dynamics of the corresponding exclusive B_s decays thereby allowing tests of the factorization hypothesis.

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1 Introduction

Some time ago it was pointed out in [1] that a clean measurement of the angle γ of the usual "non-squashed" unitarity triangle [2] of the Cabibbo–Kobayashi–Maskawa matrix (CKM matrix) [3] is possible by studying the time dependence of the color-allowed decays $\overset{(-)}{B_s} \to D_s^\pm K^\mp$. A similar analysis of the color-suppressed modes $\overset{(-)}{B_s} \to \overset{(-)}{D^0} \phi$ provides in principle also clean information about γ [4]. Because current detectors have difficulties in observing the soft photon in $D_s^* \to D_s \gamma$ decays, Aleksan, Le Yaouanc, Oliver, Pene and Raynal employed several plausible assumptions to show that the CKM angle γ can still be extracted from partially reconstructed B_s modes where that soft photon could be missed [5].

Unfortunately in all these strategies *tagging*, i.e. the distinction between initially present B_s and $\overline{B_s}$ mesons, is essential. Moreover one has to resolve the rapid $B_s - \overline{B_s}$ oscillations, which may arise from the expected large mass difference $\Delta m \equiv m_H - m_L > 0$ between the mass eigenstates B_s^H ("heavy") and B_s^L ("light") [6]. This is a formidable experimental task. In a recent paper [7] these methods have been re-considered in light of the expected perceptible lifetime difference [8] between B_s^H and B_s^L. There it has been shown that the rapid $\Delta m t$–oscillations cancel in *untagged* data samples. Whereas the extraction of γ from untagged $\overset{(-)}{B_s} \to D_s^\pm K^\mp$ requires some mild additional theoretical input, it does not require any theory beyond the validity of the CKM model from untagged $\overset{(-)}{B_s} \to \overset{(-)}{D^0} \phi$ decays [7].

In a recent publication [9] we have investigated quasi two body modes $B_s \to X_1 X_2$ into admixtures of different CP eigenstates where both X_1 and X_2 carry spin and continue to decay through CP-conserving interactions. The time-dependent angular distributions for the *untagged* decays $B_s \to D_s^{*+} D_s^{*-}$ and $B_s \to J/\psi \phi$ determine the Wolfenstein parameter η [10]. If one uses $|V_{ub}|/|V_{cb}|$ as an additional input, the CKM angle γ can be fixed. That input allows, however, also the determination of η (or γ) from the mixing-induced CP asymmetry of $B_d \to J/\psi K_S$ measuring $\sin 2\beta$ (β is another angle of the unitarity triangle [2]). Comparing these two results for η (or γ) obtained from B_s and B_d modes, respectively, an interesting test whether the $B_s - \overline{B_s}$ and $B_d - \overline{B_d}$ mixing phases are described by the Standard Model or receive additional contributions from "New Physics" can be preformed. Another application of the formalism developed in [9] is the point that a determination of γ is possible by using the $SU(2)$ isospin symmetry of strong interactions to relate *untagged* data samples of $B_s \to K^{*+} K^{*-}$ and $B_s \to K^{*0} \overline{K^{*0}}$.

Having all these results in mind it is quite natural to ask what can be learned from time-dependent *untagged* measurements of the angular distributions for $\overset{(-)}{B_s} \to D_s^{*\pm} K^{*\mp}$,

$D_{s1}(2536)^{\pm}K^{*\mp}$, $D_s^{**\pm}K^{*\mp}$ and $\overset{(-)}{B_s}\to\overset{(-)}{D^{*0}}\phi$, $\overset{(-)}{D_1}(2420)^0\phi$, $\overset{(-)}{D^{**0}}\phi$ or – more generally – from B_s modes governed by $\bar b \to \bar c u \bar s$ quark-level transitions. Since the photon(s) in the strong or electromagnetic decays of D_s^* and D^{*0} are more difficult to detect than charged particles for generic detectors, we listed also higher resonances because of their significant all-charged final states, such as $D_{s1}(2536)^+ \to D^{*+}K^0, D^{*+} \to \pi^+ D^0$ or $D_1(2420)^0 \to D^{*+}\pi^-, D^{*+} \to \pi^+ D^0$. The $K^{*\mp}$ in the above B_s-decays can be substituted by either a strange resonance or a collection of strange resonances with common spin and parity quantum numbers.

While our note focusses on quasi two body modes where each body has a well-defined spin and parity, a complementary report discusses the effects when several resonances contribute to the final state [11]. In the former case the final states cannot be classified by their CP eigenvalues as in [9]. However, they can instead be classified by their parities. To this end linear polarization states [12] are particularly useful. As we will demonstrate in the present paper, the *untagged* angular distributions for such B_s decays may inform us in a clean way about γ, if the lifetime difference between B_s^H and B_s^L is in fact sizable. In particular we do not need any theoretical input to extract this quantity from the *untagged* data samples which exhibit in addition interesting CP-violating effects. Furthermore essentially the whole hadronization dynamics can be extracted from these angular correlations. Since, for example, the $\overset{(-)}{B_s}\to D_s^{*\pm}K^{*\mp}$, $D_{s1}(2536)^{\pm}K^{*\mp}$, $D_s^{**\pm}K^{*\mp}$ modes are color-allowed whereas the $\overset{(-)}{B_s}\to\overset{(-)}{D^{*0}}\phi$, $\overset{(-)}{D_1}(2420)^0\phi$, $\overset{(-)}{D^{**0}}\phi$ channels are color-suppressed, the factorization hypothesis [13, 14], which has some justification within the $1/N_C$–expansion [15], should work quite well in the former case and should be very questionable in the latter case [16]. Therefore we expect significant non-factorizable contributions to the angular distributions for the $\overset{(-)}{B_s}\to\overset{(-)}{D^{*0}}\phi$, $\overset{(-)}{D_1}(2420)^0\phi$, $\overset{(-)}{D^{**0}}\phi$ decays. The explicit angular distributions for some of these decays will be given in a separate publication [17]. There also appropriate weighting functions are given allowing an efficient extraction of the corresponding observables from experimental data with the help of a *moment analysis* (see [18, 19]).

Our paper is organized as follows: The time-dependences of the observables of the angular distributions are calculated in Section 2 in terms of a general notation that allows an easy comparison with the results presented in [9]. In Section 3 these time-dependences are given in terms of linear polarization states which provide a useful tool to calculate the explicit angular distributions for final state configurations having definite parities. There we demonstrate explicitly that the observables of the *untagged* angular distributions for the $\bar b \to \bar c u \bar s$ (and $\bar b \to \bar u c \bar s$) decays suffice to extract the CKM angle γ. The issue of CP violation in untagged data samples is discussed in Section 4 and the main results of our paper are summarized briefly in Section 5.

2 Calculation of the general time-evolutions

In the case of the decays considered in this paper, the transition amplitudes for the quasi two body modes $\overline{B_s} \to X_1 X_2$ and $B_s \to X_1 X_2$ can be expressed as hadronic matrix elements of low energy effective Hamiltonians having the following structures:

$$H_{\text{eff}}(\overline{B_s} \to X_1 X_2) = \frac{G_{\text{F}}}{\sqrt{2}} \overline{v} \left[C_1(\mu)\overline{O}_1 + C_2(\mu)\overline{O}_2 \right] \tag{1}$$

$$H_{\text{eff}}(B_s \to X_1 X_2) = \frac{G_{\text{F}}}{\sqrt{2}} v^* \left[C_1(\mu)O_1^\dagger + C_2(\mu)O_2^\dagger \right], \tag{2}$$

where $\overline{v}$ and v denote appropriate CKM factors, $\overline{O}_k$ and O_k ($k \in \{1, 2\}$) are four-quark operators ("current-current" operators in our case) and $C_1(\mu)$ and $C_2(\mu)$ are the Wilson coefficient functions of these operators. They can be calculated perturbatively and contain the whole short distance dynamics. As usual $\mu = \mathcal{O}(m_b)$ is a renormalization scale. To be definite, for $X_1 X_2 \in \{D_s^{*+} K^{*-}, D_{s1}(2536)^+ K^{*-}, D_s^{**+} K^{*-}, D^{*0}\phi, D_1(2420)^0\phi, D^{**0}\phi\}$ we have

$$\begin{aligned}
\overline{O}_1 &= (\bar{s}_\alpha u_\beta)_{\text{V-A}} (\bar{c}_\beta b_\alpha)_{\text{V-A}} \\
\overline{O}_2 &= (\bar{s}_\alpha u_\alpha)_{\text{V-A}} (\bar{c}_\beta b_\beta)_{\text{V-A}}
\end{aligned} \tag{3}$$

$$\begin{aligned}
O_1 &= (\bar{s}_\alpha c_\beta)_{\text{V-A}} (\bar{u}_\beta b_\alpha)_{\text{V-A}} \\
O_2 &= (\bar{s}_\alpha c_\alpha)_{\text{V-A}} (\bar{u}_\beta b_\beta)_{\text{V-A}},
\end{aligned} \tag{4}$$

where the greek indices denote $SU(3)_{\text{C}}$ color indices, and the CKM factors are given by

$$\begin{aligned}
\overline{v} &= V_{us}^* V_{cb} \\
v &= V_{cs}^* V_{ub}.
\end{aligned} \tag{5}$$

Nowadays the Wilson coefficients $C_1(\mu)$ and $C_2(\mu)$ are available beyond the leading logarithmic approximation [20, 21]. A nice review of such next-to-leading order calculations has been given recently in [22], and we refer the reader to that publication for the details of such calculations.

Applying a similar notation as in [9], we obtain the following transition amplitudes for decays of B_s and $\overline{B_s}$ mesons into a configuration f of the quasi two body state $X_1 X_2$, where f is a label that defines the relative polarizations of the two hadrons X_1 and X_2:

$$\overline{A}_f \equiv \langle (X_1 X_2)_f | H_{\text{eff}}(\overline{B_s} \to X_1 X_2) | \overline{B_s} \rangle = \frac{G_{\text{F}}}{\sqrt{2}} \overline{v} \, \overline{M}_f \tag{6}$$

$$A_f \equiv \langle (X_1 X_2)_f | H_{\text{eff}}(B_s \to X_1 X_2) | B_s \rangle = \eta_{\text{P}}^f \, e^{i\phi_{\text{CP}}(B_s)} \frac{G_{\text{F}}}{\sqrt{2}} v^* M_f \tag{7}$$

with

$$\overline{M}_f \equiv \langle (X_1 X_2)_f | C_1(\mu)\overline{O}_1 + C_2(\mu)\overline{O}_2 | \overline{B_s} \rangle \tag{8}$$

$$M_f \equiv \langle (X_1 X_2)_f^{\text{c}} | C_1(\mu)O_1 + C_2(\mu)O_2 | \overline{B_s} \rangle. \tag{9}$$

In order to evaluate (7) we have performed the CP transformations

$$
\begin{aligned}
\langle (X_1\,X_2)_f | & C_1(\mu) O_1^\dagger + C_2(\mu) O_2^\dagger | B_s \rangle \\
&= \langle (X_1\,X_2)_f | (\mathcal{CP})^\dagger (\mathcal{CP}) \left[C_1(\mu) O_1^\dagger + C_2(\mu) O_2^\dagger \right] (\mathcal{CP})^\dagger (\mathcal{CP}) | B_s \rangle \\
&= \eta_P^f \, e^{i\phi_{\rm CP}(B_s)} \, \langle (X_1\,X_2)_f^{\rm C} | C_1(\mu) O_1 + C_2(\mu) O_2 | \overline{B_s} \rangle
\end{aligned}
\tag{10}
$$

by taking into account the relations

$$
(\mathcal{CP}) O_k^\dagger (\mathcal{CP})^\dagger = O_k
\tag{11}
$$

and

$$
(\mathcal{CP}) | B_s \rangle = e^{i\phi_{\rm CP}(B_s)} \, | \overline{B_s} \rangle
\tag{12}
$$

$$
(\mathcal{CP}) | (X_1\,X_2)_f \rangle = \eta_P^f \, | (X_1\,X_2)_f^{\rm C} \rangle.
\tag{13}
$$

Here $\phi_{\rm CP}(B_s)$ parametrizes the applied CP phase convention and $\eta_P^f \in \{-1, +1\}$ denotes the parity eigenvalues of the configurations f of $X_1\,X_2$. In terms of linear polarization amplitudes [12] (see also [23]) we have $\eta_P^0 = \eta_P^{\parallel} = +1$ and $\eta_P^{\perp} = -1$ for $X_1\,X_2 \in \{D_s^{*+} K^{*-}, D^{*0}\phi\}$. In contrast, for $X_1\,X_2 \in \{D_{s1}(2536)^+ K^{*-}, D_1(2420)^0\phi\}$ we have $\eta_P^0 = \eta_P^{\parallel} = -1$ and $\eta_P^{\perp} = +1$.

Let us now consider the $\overline{B_s}$ and B_s decays into the charge-conjugate quasi two body states $(X_1\,X_2)^{\rm C}$. In the case relevant for the present paper corresponding to $X_1\,X_2 \in \{D_s^{*+} K^{*-}, D_{s1}(2536)^+ K^{*-}, D_s^{***+} K^{*-}, D^{*0}\phi, D_1(2420)^0\phi, D^{**0}\phi\}$ we have $(X_1\,X_2)^{\rm C} \in \{D_s^{*-} K^{*+}, D_{s1}(2536)^- K^{*+}, D_s^{***-} K^{*+}, \overline{D}^{*0}\phi, \overline{D}_1(2420)^0\phi, \overline{D}^{**0}\phi\}$, respectively. If the charge-conjugate states are present in a configuration f with parity eigenvalue η_P^f, a similar calculation as sketched above yields

$$
\overline{A}_f^{\rm C} \equiv \langle (X_1\,X_2)_f^{\rm C} | H_{\rm eff} \left(\overline{B_s} \to (X_1\,X_2)^{\rm C} \right) | \overline{B_s} \rangle = \frac{G_{\rm F}}{\sqrt{2}} v \, M_f
\tag{14}
$$

$$
A_f^{\rm C} \equiv \langle (X_1\,X_2)_f^{\rm C} | H_{\rm eff} \left(B_s \to (X_1\,X_2)^{\rm C} \right) | B_s \rangle = \eta_P^f \, e^{i\phi_{\rm CP}(B_s)} \frac{G_{\rm F}}{\sqrt{2}} \overline{v}^* \, \overline{M}_f.
\tag{15}
$$

Using these results and the well-known formalism describing $B_s - \overline{B_s}$ mixing [7, 24], we obtain the following expressions for initially, i.e. at $t = 0$, present B_s and $\overline{B_s}$ mesons:

$$
\begin{aligned}
A_{\tilde{f}}^*(t)\, A_f(t) = {} & \frac{G_{\rm F}^2}{2} |v|^2 \, \eta_P^{\tilde{f}} \, \eta_P^f \, M_{\tilde{f}}^* \, M_f \\
& \times \left[|g_+(t)|^2 + \eta_P^{\tilde{f}} \lambda_{\tilde{f}}^* \, g_+(t)\, g_-^*(t) + \eta_P^f \lambda_f \, g_+^*(t)\, g_-(t) + \eta_P^{\tilde{f}} \eta_P^f \lambda_{\tilde{f}}^* \lambda_f \, |g_-(t)|^2 \right]
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
\overline{A}_{\tilde{f}}^*(t)\, \overline{A}_f(t) = {} & \frac{G_{\rm F}^2}{2} |v|^2 \, \eta_P^{\tilde{f}} \, \eta_P^f \, M_{\tilde{f}}^* \, M_f \\
& \times \left[|g_-(t)|^2 + \eta_P^{\tilde{f}} \lambda_{\tilde{f}}^* \, g_+^*(t)\, g_-(t) + \eta_P^f \lambda_f \, g_+(t)\, g_-^*(t) + \eta_P^{\tilde{f}} \eta_P^f \lambda_{\tilde{f}}^* \lambda_f \, |g_+(t)|^2 \right]
\end{aligned}
\tag{17}
$$

4

$$A_{\tilde{f}}^{\text{C}*}(t)\, A_f^{\text{C}}(t) = \frac{G_{\text{F}}^2}{2}\,|v|^2\, M_{\tilde{f}}^*\, M_f \tag{18}$$
$$\times \left[|g_-(t)|^2 + \eta_{\text{P}}^{\tilde{f}}\, \lambda_{\tilde{f}}^{\text{C}*}\, g_+^*(t)\, g_-(t) + \eta_{\text{P}}^{f}\, \lambda_{f}^{\text{C}}\, g_+(t)\, g_-^*(t) + \eta_{\text{P}}^{\tilde{f}}\, \eta_{\text{P}}^{f}\, \lambda_{\tilde{f}}^{\text{C}*}\, \lambda_{f}^{\text{C}}\, |g_+(t)|^2 \right]$$

$$\overline{A}_{\tilde{f}}^{\text{C}*}(t)\, \overline{A}_f^{\text{C}}(t) = \frac{G_{\text{F}}^2}{2}\,|v|^2\, M_{\tilde{f}}^*\, M_f \tag{19}$$
$$\times \left[|g_+(t)|^2 + \eta_{\text{P}}^{\tilde{f}}\, \lambda_{\tilde{f}}^{\text{C}*}\, g_+(t)\, g_-^*(t) + \eta_{\text{P}}^{f}\, \lambda_{f}^{\text{C}}\, g_+^*(t)\, g_-(t) + \eta_{\text{P}}^{\tilde{f}}\, \eta_{\text{P}}^{f}\, \lambda_{\tilde{f}}^{\text{C}*}\, \lambda_{f}^{\text{C}}\, |g_-(t)|^2 \right],$$

where

$$|g_\pm(t)|^2 \;=\; \frac{1}{4}\left[e^{-\Gamma_L t} + e^{-\Gamma_H t} \pm 2 e^{-\Gamma t}\cos(\Delta m t) \right] \tag{20}$$

$$g_+(t) g_-^*(t) \;=\; \frac{1}{4}\left[e^{-\Gamma_L t} - e^{-\Gamma_H t} - 2 i e^{-\Gamma t}\sin(\Delta m t) \right] \tag{21}$$

with $\Gamma \equiv (\Gamma_L + \Gamma_H)/2$. The observable λ_f is defined through

$$\lambda_f \equiv -\,\eta_{\text{P}}^{f}\, e^{-i\Theta_{M_{12}}^{(s)}}\, \frac{\overline{A}_f}{A_f} \tag{22}$$

with

$$\Theta_{M_{12}}^{(s)} = \pi + 2\arg(V_{ts}^* V_{tb}) - \phi_{\text{CP}}(B_s) \tag{23}$$

denoting the phase of the off-diagonal element of the $B_s - \overline{B}_s$ mass matrix. Combining (22) with (23) and (6) and (7), we observe explicitly that the convention dependent phases $\phi_{\text{CP}}(B_s)$ cancel (as they have to!) and arrive at

$$\lambda_f = \exp\left(-2\,i\,\arg\{V_{ts}^* V_{tb}\}\right)\, \frac{\overline{v}}{v^*}\, \frac{\overline{M}_f}{M_f}. \tag{24}$$

Correspondingly we have introduced

$$\lambda_f^{\text{C}} \equiv -\frac{1}{\eta_{\text{P}}^{f}\, e^{-i\Theta_{M_{12}}^{(s)}}}\, \frac{A_f^{\text{C}}}{\overline{A}_f^{\text{C}}} = \left[\exp\left(-2\,i\,\arg\{V_{ts}^* V_{tb}\}\right)\, \frac{\overline{v}}{v^*} \right]^* \frac{\overline{M}_f}{M_f}. \tag{25}$$

Note that $\lambda_{\tilde{f}}$ and $\lambda_{\tilde{f}}^{\text{C}}$ can be obtained easily from (24) and (25) by replacing f with $\tilde{f}$.

Real or imaginary parts of bilinear combinations of decay amplitudes like those given in (16)-(19) govern the angular distributions for the decay products of X_1 and X_2. In this paper we are focussing on *untagged* angular distributions, where one does not distinguish between initially present B_s and $\overline{B}_s$ mesons. The corresponding observables for $\overset{(-)}{B}_s \to X_1 X_2$ and $\overset{(-)}{B}_s \to (X_1 X_2)^{\text{C}}$ are related to real or imaginary parts of

$$\left[A_{\tilde{f}}^*(t)\, A_f(t) \right] \equiv \overline{A}_{\tilde{f}}^*(t)\, \overline{A}_f(t) + A_{\tilde{f}}^*(t)\, A_f(t) = \frac{G_{\text{F}}^2}{4}\,|v|^2\, \eta_{\text{P}}^{\tilde{f}}\, \eta_{\text{P}}^{f}\, M_{\tilde{f}}^*\, M_f \tag{26}$$
$$\times \left[\left(1 + \eta_{\text{P}}^{\tilde{f}}\, \eta_{\text{P}}^{f}\, \lambda_{\tilde{f}}^*\, \lambda_f \right) \left(e^{-\Gamma_L t} + e^{-\Gamma_H t} \right) + \left(\eta_{\text{P}}^{\tilde{f}}\, \lambda_{\tilde{f}}^* + \eta_{\text{P}}^{f}\, \lambda_f \right) \left(e^{-\Gamma_L t} - e^{-\Gamma_H t} \right) \right]$$

and

$$\left[A_f^{\text{C}*}(t) \, A_f^{\text{C}}(t) \right] \equiv \overline{A}_f^{\text{C}*}(t) \, \overline{A}_f^{\text{C}}(t) + A_f^{\text{C}*}(t) \, A_f^{\text{C}}(t) = \frac{G_{\text{F}}^2}{4} \, |v|^2 \, M_f^* \, M_f \tag{27}$$
$$\times \left[\left(1 + \eta_{\text{P}}^{\tilde{f}} \, \eta_{\text{P}}^{f} \, \lambda_{\tilde{f}}^{\text{C}*} \, \lambda_f^{\text{C}} \right) \left(e^{-\Gamma_L t} + e^{-\Gamma_H t} \right) + \left(\eta_{\text{P}}^{\tilde{f}} \, \lambda_{\tilde{f}}^{\text{C}*} + \eta_{\text{P}}^{f} \, \lambda_f^{\text{C}} \right) \left(e^{-\Gamma_L t} - e^{-\Gamma_H t} \right) \right],$$

respectively. In order to evaluate these equations we have combined (16)-(19) with the explicit time-dependences of (20) and (21). At present such *untagged* studies are obviously much more efficient from an experimental point of view than tagged analyses. In the distant future it will be feasible to collect also *tagged* data samples of B_s decays and to resolve the rapid oscillatory $\Delta m t$–terms. The corresponding tagged observables are given in (16)-(19).

Let us after these general considerations become more specific in the following section. There we give the time-evolutions in terms of linear polarization states and demonstrate that the *untagged* observables evolving as real or imaginary parts of (26) and (27) suffice to extract the CKM angle γ.

3 The extraction of the CKM angle γ

Since it is convenient to give the angular distributions in terms of the linear polarization states $f \in \{ 0 \, , \, \| \, , \, \perp \}$ (see [12, 23]), let us summarize the corresponding time-dependences in this section. The linear polarization states are characterized by the parity eigenvalues η_{P}^{f}. If we introduce the quantity

$$R_f \equiv |R_f| e^{i\rho_f} \equiv \frac{|\overline{v}|}{|v|} \frac{\overline{M}_f}{M_f}, \tag{28}$$

where ρ_f is a CP-conserving strong phase originating from strong final state interaction processes, we have in our specific case $X_1 \, X_2 \in \{ D_s^{*+} K^{*-}, \, D_{s1}(2536)^+ K^{*-}, \, D_s^{**+} K^{*-}, \, D^{*0}\phi, \, D_1(2420)^0\phi, \, D^{**0}\phi \}$

$$\lambda_f = e^{-i\gamma} R_f \tag{29}$$
$$\lambda_f^{\text{C}} = e^{+i\gamma} R_f, \tag{30}$$

where γ is the notoriously difficult to measure CKM angle of the unitarity triangle [2]. Using (5) and the Wolfenstein expansion [10] of the CKM matrix by neglecting terms of $\mathcal{O}(\lambda^2)$, where $\lambda = \sin\theta_{\text{C}} = 0.22$ is related to the Cabibbo angle, we obtain

$$R_f = \frac{1}{R_b} \frac{\overline{M}_f}{M_f} \tag{31}$$

with

$$R_b \equiv \frac{1}{\lambda}\frac{|V_{ub}|}{|V_{cb}|}. \tag{32}$$

The CKM factor R_b is constrained by present experimental data to lie within the range $R_b = 0.36 \pm 0.08$ [25, 26, 27].

If we express the hadronic matrix elements M_f defined by (9) in the form

$$M_f = |M_f|e^{i\vartheta_f}, \tag{33}$$

where ϑ_f denotes a CP-conserving strong phase shift, the time-dependent *untagged* observables corresponding to the linear polarization states [12] are in the case of $\overset{(-)}{B_s}\to X_1\,X_2$ given by

$$\begin{aligned}
\left[|A_0(t)|^2\right] = {}& \frac{G_F^2}{4}|V_{ub}V_{cs}|^2|M_0|^2 \tag{34}\\
& \times \left[\left(1+|R_0|^2\right)\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)+2|R_0|\cos(\rho_0-\gamma)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]
\end{aligned}$$

$$\begin{aligned}
\left[|A_\parallel(t)|^2\right] = {}& \frac{G_F^2}{4}|V_{ub}V_{cs}|^2|M_\parallel|^2 \tag{35}\\
& \times \left[\left(1+|R_\parallel|^2\right)\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)+2|R_\parallel|\cos(\rho_\parallel-\gamma)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]
\end{aligned}$$

$$\begin{aligned}
\left[|A_\perp(t)|^2\right] = {}& \frac{G_F^2}{4}|V_{ub}V_{cs}|^2|M_\perp|^2 \tag{36}\\
& \times \left[\left(1+|R_\perp|^2\right)\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)-2|R_\perp|\cos(\rho_\perp-\gamma)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]
\end{aligned}$$

$$\begin{aligned}
\left[A_0^*(t)A_\parallel(t)\right] = {}& \frac{G_F^2}{4}|V_{ub}V_{cs}|^2|M_0||M_\parallel|e^{i(\vartheta_\parallel-\vartheta_0)}\left[\left(1+|R_0||R_\parallel|e^{i(\rho_\parallel-\rho_0)}\right)\right. \tag{37}\\
& \left.\times\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)+\left(|R_0|e^{i(\gamma-\rho_0)}+|R_\parallel|e^{-i(\gamma-\rho_\parallel)}\right)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]
\end{aligned}$$

$$\begin{aligned}
\left[A_\parallel^*(t)A_\perp(t)\right] = {}& -\frac{G_F^2}{4}|V_{ub}V_{cs}|^2|M_\parallel||M_\perp|e^{i(\vartheta_\perp-\vartheta_\parallel)}\left[\left(1-|R_\parallel||R_\perp|e^{i(\rho_\perp-\rho_\parallel)}\right)\right. \tag{38}\\
& \left.\times\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)+\left(|R_\parallel|e^{i(\gamma-\rho_\parallel)}-|R_\perp|e^{-i(\gamma-\rho_\perp)}\right)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]
\end{aligned}$$

$$\begin{aligned}
[A_0^*(t)A_\perp(t)] = {}& -\frac{G_F^2}{4}|V_{ub}V_{cs}|^2|M_0||M_\perp|e^{i(\vartheta_\perp-\vartheta_0)}\left[\left(1-|R_0||R_\perp|e^{i(\rho_\perp-\rho_0)}\right)\right. \tag{39}\\
& \left.\times\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right)+\left(|R_0|e^{i(\gamma-\rho_0)}-|R_\perp|e^{-i(\gamma-\rho_\perp)}\right)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right].
\end{aligned}$$

For the untagged decays into the charge conjugate two body states we obtain on the other hand the following expressions:

$$\left[|A_0^{\mathrm{c}}(t)|^2\right] = \frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_0|^2 \tag{40}$$
$$\times \left[\left(1+|R_0|^2\right)\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right) + 2|R_0|\cos\left(\rho_0+\gamma\right)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]$$

$$\left[|A_\parallel^{\mathrm{c}}(t)|^2\right] = \frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_\parallel|^2 \tag{41}$$
$$\times \left[\left(1+|R_\parallel|^2\right)\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right) + 2|R_\parallel|\cos\left(\rho_\parallel+\gamma\right)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]$$

$$\left[|A_\perp^{\mathrm{c}}(t)|^2\right] = \frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_\perp|^2 \tag{42}$$
$$\times \left[\left(1+|R_\perp|^2\right)\left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right) - 2|R_\perp|\cos\left(\rho_\perp+\gamma\right)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]$$

$$\left[A_0^{\mathrm{c}*}(t)A_\parallel^{\mathrm{c}}(t)\right] = \frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_0||M_\parallel|e^{i(\vartheta_\parallel-\vartheta_0)}\left[\left(1+|R_0||R_\parallel|e^{i(\rho_\parallel-\rho_0)}\right)\right. \tag{43}$$
$$\left. \times \left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right) + \left(|R_0|e^{-i(\gamma+\rho_0)}+|R_\parallel|e^{i(\gamma+\rho_\parallel)}\right)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]$$

$$\left[A_\parallel^{\mathrm{c}*}(t)A_\perp^{\mathrm{c}}(t)\right] = \frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_\parallel||M_\perp|e^{i(\vartheta_\perp-\vartheta_\parallel)}\left[\left(1-|R_\parallel||R_\perp|e^{i(\rho_\perp-\rho_\parallel)}\right)\right. \tag{44}$$
$$\left. \times \left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right) + \left(|R_\parallel|e^{-i(\gamma+\rho_\parallel)}-|R_\perp|e^{i(\gamma+\rho_\perp)}\right)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right]$$

$$\left[A_0^{\mathrm{c}*}(t)A_\perp^{\mathrm{c}}(t)\right] = \frac{G_{\mathrm{F}}^2}{4}|V_{ub}V_{cs}|^2|M_0||M_\perp|e^{i(\vartheta_\perp-\vartheta_0)}\left[\left(1-|R_0||R_\perp|e^{i(\rho_\perp-\rho_0)}\right)\right. \tag{45}$$
$$\left. \times \left(e^{-\Gamma_L t}+e^{-\Gamma_H t}\right) + \left(|R_0|e^{-i(\gamma+\rho_0)}-|R_\perp|e^{i(\gamma+\rho_\perp)}\right)\left(e^{-\Gamma_L t}-e^{-\Gamma_H t}\right)\right].$$

Combining these equations appropriately – each of them represents a certain measurement – a determination of γ and of the strong phase shifts is possible without using any additional input. This can be seen as follows:

Let us consider the untagged observables corresponding to (34), (35) and to the real part of (37). From these rates the *ratios* of the coefficients of $e^{-\Gamma_L t} - e^{-\Gamma_H t}$ and of $e^{-\Gamma_L t} + e^{-\Gamma_H t}$ can be determined. The overall normalizations of these rates cancel in the ratios which are given by

$$u_f \equiv \frac{2|R_f|\cos\left(\rho_f-\gamma\right)}{1+|R_f|^2} \qquad (f \in \{\,0\,,\,\|\,\}) \tag{46}$$

and

$$u_{0,\parallel} \equiv \frac{|R_0|\cos\left(\vartheta_\parallel - \vartheta_0 - \rho_0 + \gamma\right) + |R_\parallel|\cos\left(\vartheta_\parallel - \vartheta_0 + \rho_\parallel - \gamma\right)}{\cos\left(\vartheta_\parallel - \vartheta_0\right) + |R_0||R_\parallel|\cos\left(\vartheta_\parallel - \vartheta_0 + \rho_\parallel - \rho_0\right)}, \tag{47}$$

respectively, and depend thus only on $|R_0|$, ρ_0, $|R_\parallel|$, $\rho_\parallel$, $\vartheta_\parallel - \vartheta_0$ and on the CKM angle γ. Using in addition the observables of the untagged B_s decays into the charge conjugate final states that are related to (40), (41) and to the real part of (43), we can determine similar ratios of the coefficients of $e^{-\Gamma_L t} - e^{-\Gamma_H t}$ and $e^{-\Gamma_L t} + e^{-\Gamma_H t}$. These charge conjugate ratios, which are given by

$$u_f^{\text{C}} \equiv \frac{2|R_f|\cos\left(\rho_f + \gamma\right)}{1 + |R_f|^2} \qquad (f \in \{\, 0\,,\, \parallel \,\}) \tag{48}$$

and

$$u_{0,\parallel}^{\text{C}} \equiv \frac{|R_0|\cos\left(\vartheta_\parallel - \vartheta_0 - \rho_0 - \gamma\right) + |R_\parallel|\cos\left(\vartheta_\parallel - \vartheta_0 + \rho_\parallel + \gamma\right)}{\cos\left(\vartheta_\parallel - \vartheta_0\right) + |R_0||R_\parallel|\cos\left(\vartheta_\parallel - \vartheta_0 + \rho_\parallel - \rho_0\right)}, \tag{49}$$

respectively, depend on the same six "unknowns" as (46) and (47) determined from (34), (35) and (37). We have therefore six observables at our disposal to determine the six "unknowns" $|R_0|$, ρ_0, $|R_\parallel|$, $\rho_\parallel$, $\vartheta_\parallel - \vartheta_0$, γ. In particular we are in a position to extract the CKM angle γ. Using furthermore the observables we have not considered so far, certain discrete ambiguities are resolved and also $|R_\perp|$, $\rho_\perp$, $\vartheta_\perp - \vartheta_0$ can be determined. Note that the overall normalizations of the rates corresponding to (34)-(45) inform us about $|V_{ub}V_{cs}| \cdot |M_f|$, where $f \in \{\, 0\,,\, \parallel\,,\, \perp \,\}$.

Obviously the major goal of this approach is the extraction of the CKM angle γ. However, also the the quantities $|R_f|$ and the strong phase shifts ρ_f, ϑ_f are of interest, since they allow insights into the hadronization dynamics of the corresponding four-quark operators.

4   CP violation

There are many CP-violating observables that can be constructed from tagged time-dependent measurements. Some of them survive even when only untagged data samples are used. The most striking *untagged* CP-violating observable is

$$\text{Im}\left\{[A_f^*(t)\,A_\perp(t)]\right\} + \text{Im}\left\{[A_f^{\text{C}*}(t)\,A_\perp^{\text{C}}(t)]\right\} = -\frac{G_{\text{F}}^2}{2}|V_{ub}V_{cs}|^2|M_f||M_\perp|$$

$$\times \left\{|R_f|\cos\left(\rho_f + \vartheta_f - \vartheta_\perp\right) + |R_\perp|\cos\left(\rho_\perp + \vartheta_\perp - \vartheta_f\right)\right\}\left(e^{-\Gamma_L t} - e^{-\Gamma_H t}\right)\sin\gamma, \tag{50}$$

where $f \in \{\, 0\,,\, \parallel \,\}$. Note that the plus sign on the l.h.s. of that equation is due to the fact that the parity eigenvalues of the final state configurations f and $\perp$ arising in the "mixed" combinations are different. The CP observable (50) is proportional to $\sin\gamma$ and occurs

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even when all strong phase shifts vanish. This CP-violating effect can be potentially very large as can be seen by employing the factorization assumption which implies vanishing strong phase shifts.

In contrast, to observe CP violation in the *untagged* interference term involving final state configurations with equal parity eigenvalues requires non-vanishing strong phase shifts as can be seen from the corresponding CP-violating observable

$$\text{Re}\left\{[A_0^*(t)\,A_\|(t)]\right\} - \text{Re}\left\{[A_0^{\text{C}*}(t)\,A_\|^{\text{C}}(t)]\right\} = \frac{G_{\text{F}}^2}{2}|V_{ub}V_{cs}|^2|M_0||M_\||$$
$$\times\left\{|R_0|\sin(\rho_0+\vartheta_0-\vartheta_\|) + |R_\||\sin(\rho_\|+\vartheta_\|-\vartheta_0)\right\}\left(e^{-\Gamma_L t} - e^{-\Gamma_H t}\right)\sin\gamma. \quad (51)$$

The last category of CP-violating effects in *untagged* data samples is related to

$$\left[|A_f(t)|^2\right] - \left[|A_f^{\text{C}}(t)|^2\right]$$
$$= \eta_{\text{P}}^f G_{\text{F}}^2|V_{ub}V_{cs}|^2|M_f|^2|R_f|\sin\rho_f\left(e^{-\Gamma_L t} - e^{-\Gamma_H t}\right)\sin\gamma \quad (52)$$

with $f \in \{0\,,\,\|\,,\,\perp\}$ and requires also non-vanishing strong phase shifts. This last category is the only one that has been considered so far in the literature [7].

5 Summary

We have calculated the time-dependences of the observables of angular distributions for B_s decays caused by $\bar{b} \to \bar{c}u\bar{s}$ quark-level transitions both in a general notation and in terms of linear polarization states. Examples for exclusive modes belonging to this decay category are the color-allowed and color-suppressed channels $\overset{(-)}{B_s} \to D_s^{*\pm}K^{*\mp}$, $D_{s1}(2536)^{\pm}K^{*\mp}$, $D_s^{**\pm}K^{*\mp}$ and $\overset{(-)}{B_s} \to \overset{(-)}{D^{*0}}\phi$, $\overset{(-)}{D_1}(2420)^0\phi$, $\overset{(-)}{D^{**0}}\phi$, respectively. Since charged particles are easier to detect for generic detectors than the photon(s) in the strong or electromagnetic decays of D_s^* and D^{*0}, we have also listed higher resonances exhibiting significant all-charged final states. The information that is provided by the corresponding angular correlations allows – without any theoretical input – the extraction both of the notoriously difficult to measure CKM angle γ and of the whole hadronization dynamics of these decays thereby allowing e.g. tests of the factorization hypothesis.

If the lifetime difference between the B_s mass eigenstates B_s^L and B_s^H is sizable, as is indicated by certain present theoretical analyses, even *untagged* B_s data samples suffice to accomplish this ambitious task. Interestingly, some of the many CP-violating observables that can be constructed from tagged measurements survive also in that *untagged* case and are potentially very large. One class of these untagged CP-violating observables is proportional to $\sin\gamma$ and arises even when all strong phase shifts vanish.

From an experimental point of view, untagged analyses of B_s-meson decays are obviously much more efficient than tagged studies. The feasibility of our *untagged* strategies for extracting γ in a clean way depends, however, crucially on a sizable lifetime difference of the B_s system. Even if this lifetime splitting should turn out to be too small for untagged analyses, once a non-vanishing lifetime difference has been established experimentally, the formalism presented in our paper must be used in the case of tagged measurements in order to extract γ correctly. Clearly time will tell and an exciting future may lie ahead of us.

Acknowledgments

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